

<b>Instructors:</b> 1. Dr. Rola Alseidi	 <p>Philadelphia University Faculty of Science Department of Mathematics Midterm Exam</p>	<b>Academic Year:</b> 2022-2023 <b>Semester:</b> Fall <b>Date:</b> 21/12/2022 <b>Course:</b> Real Analysis (1) <b>Duration:</b> 75 Min
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**Name:**

**I.D. Number:**

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**Question One:** [7 points ]

1. Let  $S$  be a nonempty subset of  $\mathbb{R}$  and  $S$  is bounded above. Give a necessary and sufficient condition for a number  $u \in \mathbb{R}$  to be supremum of  $S$ .

2. State the Archimedean Property.

3. Find

$$\inf\left\{\frac{1}{n} - \frac{n}{n+1}, n \in \mathbb{N}\right\}$$

**Question Two:** [6 points]

Consider the sequence

$$a_n = \frac{3n + 1}{2n + 1}, n \in \mathbb{N}$$

1. Write the first two terms of  $a_n$ .

2. Use the definition of the limit of a sequence to establish the following limit

$$\lim_{n \rightarrow \infty} \frac{3n + 1}{2n + 1} = \frac{3}{2}$$

**Question Three:** [6 points ]

Evaluate the following limits (Show the details of your work)

1.  $\lim_{n \rightarrow \infty} \frac{1}{2^n}$  .

2.  $\lim_{n \rightarrow \infty} \frac{2n}{n^2 + 1}$  .

3.  $\lim_{n \rightarrow \infty} \frac{\sin(n + 1)}{n^2 + 1}$  .

**Question four:** [6 points ]

Let  $x_1 = 8$  and  $x_{n+1} = \frac{1}{2}x_n + 2$ .

1. Show that  $x_n$  is bounded and monotone.

2. Is  $x_n$  convergent? explain. If so find its limit.

**Question five:** [5 points ]

Circle True or False. Read each statement carefully before answering.

- (a) True    False    If  $x$  and  $y$  are real numbers with  $x < y$ , there exists an irrational number  $z$ :  
 $x < z < y$
- (b) True    False    If  $a > b$  and  $c < 0$ , then  $ca > cb$ .
- (c) True    False    Every bounded sequence is convergent.
- (d) True    False    The sum of two divergent sequences diverges.
- (e) True    False    A monotone sequence of real numbers is divergent.

**Good Luck**